Harmonic Analysis Homework Sheet 1

Exercise 1.1

Let $0 and assume <math>f, (f_k)_{k \in \mathbb{N}}, g \in L^p(X, \mu)$.

1. Show that

$$\|\sum_{k=1}^{N} f_k\|_{L^p(X)} \le N^{1/p-1} \sum_{k=1}^{N} \|f_k\|_{L^p(X)}$$

holds for any $N \in \mathbb{N}$. (Hint: First show $\|\sum_{k=1}^{N} f_k\|_p^p \leq \sum_{k=1}^{N} \|f_k\|_p^p$ and then use Hölder's inequality for the sum on the right side.) Is the constant $N^{1/p-1}$ sharp?

2. If additionally $f, g \ge 0$, prove the reverse triangle inequality $||f + g||_p \ge ||f||_p + ||g||_p$.

Exercise 1.2

Show Proposition 1.1.3 (on simple properties of d_f) in the notes.

Exercise 1.3

Prove the marked items in Proposition 1.2.5 (on simple properties of f^*) in the notes.