Summer term 2020

Harmonic Analysis
Homework Sheet 1

Exercise 1.1
Let $0 < p < 1$ and assume $f, (f_k)_{k\in\mathbb{N}}, g \in L^p(X,\mu)$.

1. Show that
$$\|\sum_{k=1}^{N} f_k\|_{L^p(X)} \leq N^{1/p-1} \sum_{k=1}^{N} \|f_k\|_{L^p(X)}$$
holds for any $N \in \mathbb{N}$. (Hint: First show $\|\sum_{k=1}^{N} f_k\|_p^p \leq \sum_{k=1}^{N} \|f_k\|_p^p$ and then use Hölder's inequality for the sum on the right side.) Is the constant $N^{1/p-1}$ sharp?

2. If additionally $f, g \geq 0$, prove the reverse triangle inequality $\|f+g\|_p \geq \|f\|_p + \|g\|_p$.

Exercise 1.2
Show Proposition 1.1.3 (on simple properties of d_f) in the notes.

Exercise 1.3
Prove the marked items in Proposition 1.2.5 (on simple properties of f^*) in the notes.