Multistability of Small Zero-one Reaction Networks

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Zero-one biochemical reaction networks play key roles in cell signalling such as signalling pathways regulated by protein phosphorylation. Multistability of reaction networks is a crucial dynamics feature enabling decision-making in cells. It is well known that multistability can be lifted from a "subnetwork" (a network with less species and fewer reactions) to large networks. So, we aim to explore the multistability problem of small zero-one networks. In this work, we prove the following main results: 1. any zero-one network with a one-dimensional stoichiometric subspace admits at most one positive steady state (it must be stable), and all the one-dimensional zero-one networks can be classified according to if they indeed admit a stable positive steady state or not; 2. any two-dimensional zero-one network with up to three species either admits only degenerate positive steady states, or admits at most one positive steady state (it must be stable); 3. the smallest zero-one networks (here, by "smallest", we mean these networks contain species as few as possible) that admit nondegenerate multistationarity/multistability contain three species and five/six reactions, and they are three-dimensional. In these proofs, we use the theorems based on the Brouwer degree theory and the theory of real algebraic geometry. Moreover, applying the tools of computational real algebraic geometry, we provide a systematical way for detecting the networks that admit nondegenerate multistationarity/multistability.