Nonnegative homogeneous symmetric functions

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A symmetric function is a formal power series of bounded degree that is invariant under the permutation action of the infinite symmetric group. The ring of symmetric functions can be defined as the inverse limit of the rings of symmetric polynomials. For a fixed degree d and a sufficiently large number of variables, the degree d homogeneous components of these rings are isomorphic vector spaces.

Using the identification that maps products of power sum polynomials in different numbers of variables to each other, we study the cones of nonnegative symmetric polynomials and sums of squares. These cones form descending chains as the number of variables increases, and we are interested in their behavior in the limit.

Surprisingly, in the limit the cone of nonnegative symmetric functions is not semialgebraic. We provide explicit examples of symmetric forms that are nonnegative but not sums of squares, and we show how tropicalization techniques can be used to classify all inequalities of the form

$$\prod_{i=1}^{\ell} p_i^{a_i} \geq \prod_{i=1}^{\ell} p_i^{b a_i}$$

on the nonnegative orthant, for $a_i, b_i \in \mathbb{Z}$.

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