A regularized version of the Christoffel function over the sphere S^d

Leandro Bentancur[1], Ernesto García[2], Mauricio Velasco[3]

[1]Universidad de la República, Montevideo, Uruguay
leandrob@cmat.edu.uy

[2]Laboratory for Analysis and Architecture of Systems, Toulouse, France
ernesto.garcia-ciganda@laas.fr

[3]Universidad de la República, Montevideo, Uruguay
mvelasco@cmat.edu.uy

A basic problem in optimization and statistics is the recovery of a measure from a given collection of moments. The Christoffel function (and its reproducing Christoffel–Darboux kernel) is a classical tool from approximation theory and orthogonal polynomials, which can be used to approach this task. The Christoffel function allows us to obtain an approximation for the density of the measure as an explicit rational function involving the inverse of the given truncated moment matrix M_n.

Building on recent work by Lasserre, we introduce a novel family of regularized Christoffel functions on spheres to improve their quality as density estimators. Our formulation provides an explicit sum-of-squares (SOS) representation of the reciprocal Christoffel function, which facilitates both efficient computation and theoretical analysis. We also prove quantitative error bounds for density recovery using regularized Christoffel functions.

Finally we explore applications of this approach to the estimation of occupation measures for diffusions and also to estimate quasi-stationary distributions for certain stochastic processes using data obtained exclusively from simulations.