Adding inequality constraints for one-shot methods

Lea Fischer[1*], Nicolas R. Gauger[1], Lisa Kusch[2]

[1] University of Kaiserslautern-Landau (RPTU), Chair for Scientific Computing
[2] Eindhoven University of Technology, Computational Illumination Optics Group
[*] lea.fischer@rptu.de

The one-shot method is an optimization method typically used in design optimization problems with partial differential equations where the underlying state equation is treated by numerically expensive fixed-point solves.

In [1] the authors extended the one-shot method by incorporating additional equality constraints. Now the time has come to include inequality constraints. This widens the area of application by allowing inequality constraints additional to the state equation given by partial differential equations and makes it possible to use the one-shot method in a hybrid strategy for multi-objective optimization. The procedure is analogous to that in [2]. The one-shot step is derived using the Lagrangian. Then an exact penalty function of doubly augmented Lagrangian type is constructed and its analysis is used to derive suitable preconditioners. Here the challenge arises that the doubly augmented Lagrangian of an inequality constrained optimization problem adds another minimization step over an additional slack variable. This is overcome by solving this minimization problem explicitly which in turn leads to an exact penalty function that is not twice continuously differentiable. As a result the concept of second-order subdifferentials is explored.

Future research might incorporate the resulting one-shot method with additional inequality constraints into multiobjective optimization using a hybrid strategy. Especially in problems with a non-convex Pareto front this could be beneficial compared to the one-shot method with additional equality constraints.

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References

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