Lissajous Varieties

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We study the Zariski closure of the image of an affine-linear space under the componentwise cosine map. We refer to this algebraic object as a Lissajous variety, which is rational and irreducible. Its degree equals the volume of a polytope, up to a combinatorial factor, and its defining equations arise from rank constraints on a matrix with polynomial entries.

This variety finds an application in the Kuramoto model, which is one of the most investigated models in the context of coupled oscillators. Specifically, studying the steady states of the Kuramoto model amounts to studying the intersection of a Lissajous variety with an affine linear space of complementary dimension. In this setting, the variety is defined from the incidence matrix of the graph representing the oscillator network and is parametrized by sines. Moreover, we can interpret this framework from an optimization perspective. We define the positive part of the Lissajous variety parametrized by sines and show that its intersection with the affine-linear space is the unique minimizer of a convex optimization problem.

This is joint work with Simon Telen.