Software for Generating Points Uniformly in a Convex Polytope

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We present a software package implementing a new algorithm for generating points uniformly in a convex polytope P in $\mathbb{R}^d$ defined by a system of linear inequalities. The package can be applied for solving problems in several areas of science and engineering via Monte Carlo methods.

The algorithm is based on splitting P into small pieces (cover units), selecting one of them with probability determined by its volume, and generating a point in the chosen unit. Most of cover units are congruent boxes of volume V which is a factor of the volume of P. Most of the remaining non-box cover units are polytopes with $2d$ facets, the generation of points in which is simple and fast. The algorithm requires the initialization of three tables. Table 1 contains coordinates of a vertex of each of the box cover units. Table 2 contains coefficients of linear inequalities defining each of the non-box cover units and coordinates of two points defining the bounding box for the unit. Table 3 contains the volumes of all the non-box units.

Most software routines developed to date for generating points in low-dimensional polytopes are based on decomposition of a convex polytope into simplices (triangulation), random selection of one of them, and generating a point uniformly in the selected simplex. The performance of the presented algorithm is superior to triangulation algorithms in terms of generation speed since generating a point in a box is simpler and faster than generation of a point in a simplex and since congruence of the boxes enables one to select one of them more conveniently. The presented algorithm is similar to very fast grid algorithm for generating points uniformly in a compact set in $\mathbb{R}^d$ developed by Devroye [1], though unlike the Devroye algorithm, it does not belong to the class of rejection algorithms. The Devroye algorithm is not designed specifically for sampling from a polytope and software implementing this algorithm or a similar algorithm for generating points in convex polytopes has not been developed to date.

The software package implementing our algorithm is programmed in C++ and includes three main routines. All of them receive as an input an object that realizes a system of inequalities of a polytope. The first routine calculates the volume of a convex polytope. The second one creates two vectors defining the bounding box for a convex polytope. The third routine removes redundant inequalities from a system of inequalities defining a convex polytope.

1. Devroye, L.: Grid Methods in Simulation and Random Variate Generation. Computing 37, 71–84 (1986)